

Quantization of The Electroweak Theory in The Hamiltonian Path-Integral Formalism

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The quantization of the $SU(2) \times U(1)$ gauge-symmetric electroweak theory is performed in the Hamiltonian path-integral formalism. In this quantization, we start from the Lagrangian given in the unitary gauge in which the unphysical Goldstone fields are absent, but the unphysical longitudinal components of the gauge fields still exist. In order to eliminate the longitudinal components, it is necessary to introduce the Lorentz gauge conditions as constraints. These constraints may be incorporated into the Lagrangian by the Lagrange undetermined multiplier method. In this way, it is found that every component of a four-dimensional vector potential has a conjugate counterpart. Thus, a Lorentz-covariant quantization in the Hamiltonian path-integral formalism can be well accomplished and leads to a result which is the same as given by the Faddeev-Popov approach of quantization.

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In our preceding paper^[1], the quantum electroweak theory without involving the Goldstone bosons^[2,3] was established starting from the Lagrangian given in the unitary gauge by the Faddeev-Popov approach^[4] in the Lagrangian path-integral formalism. The quantum theory given in the α -gauge shows good renormalizability. The unitarity is ensured by the limiting procedure: $\alpha \rightarrow \infty$ by which the results calculated in the α -gauge will be converted to the physical ones as should be obtained in the unitary gauge. An important feature of the theory is that the theory established is still of $SU(2) \times U(1)$ gauge symmetry. To check this point, in this paper, the quantization of the theory will be carried out in the Hamiltonian path-integral formalism^[5-7]. This quantization, unlike the quantization by the Faddeev-Popov approach, does not concern the gauge transformation and the gauge-invariance of the Lagrangian chosen for the quantization.

For simplicity, we limit ourself to discuss the electroweak interaction system for one generation of leptons. the Lagrangian is^[2,3,8]

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_\phi \quad (1)$$

where

$$\mathcal{L}_g = -\frac{1}{4}F^{\alpha\mu\nu}F_{\mu\nu}^\alpha \quad (2)$$

$$\mathcal{L}_f = \bar{L}i\gamma^\mu D_\mu L + \bar{l}_R i\gamma^\mu D_\mu l_R \quad (3)$$

and

$$\mathcal{L}_\phi = \frac{1}{2}(D^\mu \phi)^+(D_\mu \phi) - \frac{1}{2}\mu^2 \phi^+ \phi - \frac{1}{4}\lambda(\phi^+ \phi)^2 - \frac{f_l}{\sqrt{2}}(\bar{L}\phi l_R + \bar{l}_R \phi^+ L) \quad (4)$$

are respectively the parts of the Lagrangian for the gauge boson, lepton and scalar particle fields. In the above,

$$F_{\mu\nu}^\alpha = \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha + g\varepsilon^{\alpha\beta\gamma}A_\mu^\beta A_\nu^\gamma \quad (5)$$

where $\alpha = 0, 1, 2, 3$, $A_\mu^i = (A_\mu^a, B_\mu)$, A_μ^a and B_μ are the $SU(2)$ and $U(1)$ gauge fields respectively and the Levi-Civita tensor is defined by

$$\varepsilon^{\alpha\beta\gamma} = \begin{cases} \epsilon^{abc}, & \text{if } \alpha, \beta, \gamma = a, b, c = 1, 2 \text{ or } 3; \\ 0, & \text{if } \alpha, \beta \text{ and/or } \gamma = 0 \end{cases} \quad (6)$$

$$L = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix} \quad (7)$$

here ν_L represents the left handed neutrino field and l_L the left-handed lepton field, l_R is the right-handed lepton field ,

$$D_\mu = \partial_\mu - ig \frac{\tau^a}{2} A_\mu^a - ig' \frac{Y}{2} B_\mu \quad (8)$$

is the covariant derivative in which $\frac{\tau^a}{2}$ and $\frac{Y}{2}$ are the generators of $SU(2)_T$ and $U(1)_Y$ gauge groups, respectively, and

$$\phi(x) = \begin{pmatrix} 0 \\ H(x) + v \end{pmatrix} \quad (9)$$

is the scalar field doublet, g, g', μ^2, λ and f_l are the coupling constants with a relation $v = \sqrt{-\mu^2/\lambda}$.

In above Lagrangian, still exist the unphysical longitudinal parts of the gauge fields. They may completely be eliminated by introducing the Lorentz conditions

$$\chi^\alpha \equiv \partial^\mu A_\mu^\alpha = 0 \quad (10)$$

According to the general procedure of dealing with constrained systems ^[5,6], the Lorentz conditions, as constraints, may be incorporated into the Lagrangian by the Lagrange multiplier method

$$\mathcal{L}_\lambda = \mathcal{L} + \lambda^\alpha \partial^\mu A_\mu^\alpha \quad (11)$$

where \mathcal{L} is the Lagrangian shown in Eqs.(1)-(4) and λ^α are the Lagrange multipliers. The above Lagrangian will suitably be chosen to be the starting point of performing the Hamiltonian path-integral quantization. The advantage of the Lagrangian is that it provides each component of a vector potential a canonically conjugate counterpart. In fact, according to the usual definition of the canonical momentum (density) of a field variable, it is found

$$\Pi_\mu^\alpha = \frac{\partial \mathcal{L}_\lambda}{\partial \dot{A}^{\alpha\mu}} = F_{\mu 0}^\alpha + \lambda^\alpha \delta_{\mu 0} = \begin{cases} F_{k0}^\alpha \equiv E_k^\alpha, & \text{if } \mu = k = 1, 2, 3 \\ \lambda^\alpha \equiv -E_0^\alpha, & \text{if } \mu = 0 \end{cases} \quad (12)$$

here, as we see, the Lagrange multipliers act as the time-components of the conjugate momenta. With the conjugate momenta defined above and the conjugate momenta for fermion fields and the Higgs field which are defined as

$$\begin{aligned} \Pi_l &= \frac{\partial \mathcal{L}_\lambda}{\partial \dot{l}} = il^+, \quad \bar{\Pi}_l = \frac{\partial \mathcal{L}_\lambda}{\partial \dot{\bar{l}}} = 0 \\ \Pi_\nu &= \frac{\partial \mathcal{L}_\lambda}{\partial \dot{\nu}} = i\nu_L^+, \quad \bar{\Pi}_\nu = \frac{\partial \mathcal{L}_\lambda}{\partial \dot{\bar{\nu}}} = 0 \end{aligned} \quad (13)$$

and

$$\Pi_H = \frac{\partial \mathcal{L}_\lambda}{\partial \dot{H}} = \dot{H} \quad (14)$$

according to the standard procedure, the Lagrangian in Eq.(11) may readily be recast in the first-order form

$$\mathcal{L}_\lambda = E^{\alpha\mu} \dot{A}_\mu^\alpha + \Pi_l \dot{l} + \Pi_\nu \dot{\nu} + \Pi_H \dot{H} - \mathcal{H} + A_0^\alpha \varphi^\alpha - E_0^\alpha \chi^\alpha \quad (15)$$

where

$$\mathcal{H} = \mathcal{H}_g + \mathcal{H}_f + \mathcal{H}_H \quad (16)$$

is the Hamiltonian (density) in which

$$\mathcal{H}_g = \frac{1}{2}[(E_k^\alpha)^2 + (B_k^\alpha)^2] \quad (17)$$

here

$$B_k^\alpha = -\frac{1}{2}\varepsilon_{ijk}F_{ij}^\alpha \quad (18)$$

$$\mathcal{H}_f = -i\bar{\nu}_L \gamma^k \partial_k \nu_L - i\bar{l} \gamma^k \partial_k l - \frac{1}{2}\bar{L} \gamma^k (g\tau^a A_k^a - g'B_k)L + g'\bar{l}_R \gamma^k B_k l_R \quad (19)$$

and

$$\begin{aligned}\mathcal{H}_H = & \frac{1}{2}\Pi_H^2 + \frac{1}{2}(\nabla H)^2 + \frac{1}{2}\mu^2(H+v)^2 + \frac{\lambda}{4}(H+v)^4 \\ & + \frac{1}{4}\{g^2[(A_k^1)^2 + (A_k^2)^2] + (g'B_k - gA_k^3)^2\}(H+v)^2 \\ & + \frac{f_l}{\sqrt{2}}\bar{l}l(H+v)\end{aligned}\quad (20)$$

$$\varphi^\alpha = \varphi_g^\alpha + \varphi_f^\alpha + \varphi_H^\alpha \quad (21)$$

in which

$$\varphi_g^\alpha = \partial^\mu E_\mu^\alpha - g\varepsilon^{\alpha\beta\gamma}E_k^\beta A^{\gamma k} \quad (22)$$

$$\varphi_f^\alpha = \begin{cases} \frac{1}{2}g\bar{L}\gamma_0\tau^a L, & \text{if } \alpha = a = 1, 2, 3; \\ -\frac{1}{2}g'(\bar{L}\gamma_0 L + 2\bar{l}_R\gamma_0 l_R), & \text{if } \alpha = 0 \end{cases} \quad (23)$$

and

$$\varphi_H^\alpha = \begin{cases} \frac{1}{4}g(gA_0^a - \delta^{a3}g'B_0)(H+v)^2, & \text{if } \alpha = a = 1, 2, 3; \\ \frac{1}{4}g'(g'B_0 - gA_0^3)(H+v)^2, & \text{if } \alpha = 0 \end{cases} \quad (24)$$

and χ^α was defined in Eq.(10).

From the structure of the Lagrangian in Eq.(15), it is clearly seen that the last two terms in Eq.(15) are actually given by incorporating the constraint conditions

$$\varphi^\alpha = 0 \quad (25)$$

and that in Eq.(10) into the Lagrangian by the Lagrange multiplier method. These constraint conditions may be derived from the stationary condition of the action $S_\lambda = \int d^4x \mathcal{L}_\lambda(x)$. From this derivation, one may also obtain equations of motion (see Appendix A) in which there are time-derivatives of the dynamical field variables $A_k^a, B_k, F_{k0}^a = E_k^a$ and $B_{k0} = E_k^0$; whereas in Eqs.(10) and (25) there are no such derivatives. Therefore, Eqs.(10) and (25) can only be identified with the constraint equations. Since $\partial^\mu E_\mu^\alpha = \partial^\mu E_{L\mu}^\alpha$ and $\partial^\mu A_\mu^\alpha = \partial^\mu A_{L\mu}^\alpha$ where $E_{L\mu}^\alpha$ and $A_{L\mu}^\alpha$ are the longitudinal parts of the canonical variables E_μ^α and A_μ^α respectively, we see, the conditions in Eq.(26) and (27) are responsible respectively for constraining the unphysical longitudinal parts of the canonical variables $E_{L\mu}^\alpha$ and $A_{L\mu}^\alpha$. Therefore, only the transverse parts of the variables, $E_{T\mu}^\alpha$ and $A_{T\mu}^\alpha$, can be viewed as independent dynamical field variables. Because each of the transverse vectors $E_{T\mu}^\alpha$ and $A_{T\mu}^\alpha$ contains three independent components, they are sufficient to describe the polarization states of the massive gauge fields.

Let us turn to the solutions of the equations (10) and (25). The solution of equation (10), as is well-known, is

$$A_{L\mu}^\alpha = 0 \quad (26)$$

For the equation (25) with the φ^α being represented in Eqs.(21)-(24), we would like to note that the function φ_g^α in Eq.(22) can also be written in the form of Lorentz-covariance

$$\varphi_g^\alpha = \partial^\mu E_\mu^\alpha - g\varepsilon^{\alpha\beta\gamma}E_\mu^\beta A^{\gamma\mu} \quad (27)$$

This is because the added term $g\varepsilon^{\alpha\beta\gamma}E_0^\beta A_0^\gamma$ in the above gives a vanishing contribution to the Lagrangian (see the term $A_0^\alpha\varphi^\alpha$ in Eq.(15)) due to the identity $\varepsilon^{\alpha\beta\gamma}A_0^\alpha A_0^\gamma \equiv 0$. On substituting Eq.(26) into Eq.(25) and noticing that the longitudinal vector $E_{L\mu}^\alpha$ can always be represented as

$$E_{L\mu}^\alpha = \partial_\mu Q^\alpha \quad (28)$$

where Q^a is a scalar function, the equation in Eq.(25) may be written in the form^[4,5]

$$K^{\alpha\beta}(x)Q^\beta(x) = R^\alpha(x) \quad (29)$$

where

$$K^{\alpha\beta} = \delta^{\alpha\beta} \square - g\varepsilon^{\alpha\beta\gamma} A_T^{\gamma\mu} \partial_\mu \quad (30)$$

and

$$R^\alpha = g\varepsilon^{\alpha\beta\gamma} E_{T\mu}^\beta A_T^{\gamma\mu} - \varphi_f^\alpha(A_{0T}^\alpha) - \varphi_H^\alpha(A_{0T}^\alpha) \quad (31)$$

here $\varphi_f^\alpha(A_{0T}^\alpha)$ and $\varphi_H^\alpha(A_{0T}^\alpha)$ are defined in Eqs.(23) and (24) with the A_0^α being replaced by A_{0T}^α . With the aid of the Green's function $\Delta^{\alpha\beta}(x-y)$ (the ghost particle propagator) which satisfies the equation

$$K^{\alpha\gamma}(x)\Delta^{\gamma\beta}(x-y) = \delta^{\alpha\beta}\delta^4(x-y) \quad (32)$$

the solution of equation (29) is found to be

$$Q^\alpha(x) = \int d^4y \Delta^{\alpha\beta}(x-y) R^\beta(y) \quad (33)$$

Inserting this result into Eq.(28), we get

$$E_{L\mu}^\alpha = E_{L\mu}^\alpha(A_{T\mu}^\alpha, E_{T\mu}^\alpha, \dots) \quad (34)$$

which is a functional of the independent field variables $A_{T\mu}^\alpha, E_{T\mu}^\alpha$ and others.

Now, we are ready to carry out the quantization in the Hamiltonian path-integral formalism. In accordance with the basic idea of path-integral quantization^[5-7], we are allowed to directly write out an exact generating functional of Green's functions by making use of the Hamiltonian which is expressed in terms of the independent field variables

$$Z[J] = \frac{1}{N} \int D(\Pi^*, \Phi^*) \exp\{i \int d^4x (\Pi^* \cdot \dot{\Phi}^* - \mathcal{H}^*(\Pi^*, \Phi^*) + J^* \cdot \Phi^*)\} \quad (35)$$

where Φ^* and Π^* stand for all the independent variables $(A_{T\mu}^\alpha, \nu, l, H)$ and the conjugate ones $(E_{T\mu}^\alpha, \Pi_\nu, \Pi_l, \Pi_H)$ respectively, J denotes the external sources and $\mathcal{H}^*(\Pi^*, \Phi^*)$ is the Hamiltonian defined by^[6]

$$\mathcal{H}^*(\Pi^*, \Phi^*) = \mathcal{H}(\Pi, \Phi)|_{A_{L\mu}^\alpha=0, E_{L\mu}^\alpha=E_{L\mu}^\alpha(\Pi^*, \Phi^*)} \quad (36)$$

This Hamiltonian, as it stands, has a complicated functional structure which is not convenient for establishing the perturbation theory. Therefore, one still expects to represent the generating functional through the full vectors A_μ^α and E_μ^α . For this purpose, it is necessary to introduce the delta-functionals into the generating functional like this^[6]

$$\begin{aligned} Z[J] &= \frac{1}{N} \int D(\Pi, \Phi) \delta[A_L] \delta[E_L - E_L(\Pi^*, \Phi^*)] \\ &\quad \times \exp\{i \int d^4x [\Pi \cdot \dot{\Phi} - \mathcal{H}(\Pi, \Phi) + J \cdot \Phi]\} \end{aligned} \quad (37)$$

where $\Phi = (A_\mu^\alpha, \nu, l, H)$ and $\Pi = (E_\mu^\alpha, \Pi_\nu, \Pi_l, \Pi_H)$.

The delta-functionals in Eq.(37) can be expressed as a useful form as follows^[6](see appendix B)

$$\delta[A_L] \delta[E_L - E_L(\Pi^*, \Phi^*)] = \det M[A] \delta[\varphi] \delta[\chi] \quad (38)$$

where $\delta[\varphi]$ and $\delta[\chi]$ represent the constraint conditions in Eq.(10) and (25) and $M[A]$ is a matrix whose elements are given by the following Poisson bracket^[6]

$$\begin{aligned} M^{\alpha\beta}(x, y) &= \{\varphi^\alpha(x), \chi^\beta(y)\} \\ &= \int d^4z \left\{ \frac{\delta\varphi^\alpha(x)}{\delta A_\mu^\gamma(z)} \frac{\delta\chi^\beta(y)}{\delta E^{\gamma\mu}(z)} - \frac{\delta\varphi^\alpha(x)}{\delta E_\mu^\gamma(z)} \frac{\delta\chi^\beta(y)}{\delta A^{\gamma\mu}(z)} \right\} \end{aligned} \quad (39)$$

These matrix elements are easily evaluated by using the expressions denoted in Eqs.(10), (21), (27), (23) and (24). The result is

$$M^{\alpha\beta}(x, y) = \partial_x^\mu [D_\mu^{\alpha\beta}(x) \delta^4(x-y)] \quad (40)$$

where

$$D_{\mu}^{\alpha\beta}(x) = \delta^{\alpha\beta} \partial_{\mu}^x - g \varepsilon^{\alpha\beta\gamma} A_{\mu}^{\gamma}(x) \quad (41)$$

Upon inserting the relation in Eq.(38) into Eq.(37) and employing the Fourier representation for $\delta[\varphi]$

$$\delta[\varphi] = \int D\left(\frac{\eta}{2\pi}\right) e^{i \int d^4x \eta^{\alpha}(x) \varphi^{\alpha}(x)} \quad (42)$$

we have

$$\begin{aligned} Z[J] &= \frac{1}{N} \int D(\Pi, \Phi) D\left(\frac{d\eta}{2\pi}\right) \det M[A] \delta[\chi] \\ &\times \exp\left\{i \int d^4x (\Pi \cdot \dot{\Phi} + \eta^{\alpha} \varphi^{\alpha} - \mathcal{H} + J \cdot \Phi)\right\} \end{aligned} \quad (43)$$

For later convenience, the E_0^{α} -dependent terms will be extracted from the first two terms in the above exponent and thus Eq.(43) will be rewritten as

$$\begin{aligned} Z[J] &= \frac{1}{N} \int D(\Pi', \Phi') D(E_0, A_0) D\left(\frac{\eta}{2\pi}\right) \det M[A] \delta[\chi] \\ &\times \exp\left\{i \int d^4x [E_0^{\alpha} (\dot{A}_0^{\alpha} - \dot{\eta}^{\alpha}) + \Pi' \cdot \dot{\Phi}' + \eta^{\alpha} \overline{\varphi}^{\alpha} - \mathcal{H} + J \cdot \Phi]\right\} \end{aligned} \quad (44)$$

where

$$\Pi' \cdot \dot{\Phi}' = E^{\alpha k} \dot{A}_k^{\alpha} + \Pi_{\nu} \dot{\nu} + \Pi_l \dot{l} + \Pi_H \dot{H} \quad (45)$$

and

$$\eta^{\alpha} \overline{\varphi}^{\alpha} = \eta^{\alpha} (\partial^k E_k^{\alpha} + g \varepsilon^{\alpha\beta\gamma} A_k^{\beta} E^{\gamma k} + \varphi_f^{\alpha} + \varphi_H^{\alpha}) \quad (46)$$

The integral over E_0^{α} in Eq.(44) gives the delta-functional

$$\delta[\dot{A}_0^{\alpha} - \dot{\eta}^{\alpha}] = \det |\partial_0|^{-1} \delta[\eta^{\alpha} - A_0^{\alpha}] \quad (47)$$

The determinant in the above, as a constant, may be put in the normalization constant N . The delta-functional $\delta[\eta^{\alpha} - A_0^{\alpha}]$ will be used to perform the integration over η^{α} in Eq.(44). After these manipulations, we get

$$\begin{aligned} Z[J] &= \frac{1}{N} \int D(\Pi', \Phi') D(A_0) \det M[A] \delta[\chi] \\ &\times \exp\left\{i \int d^4x [\Pi' \cdot \dot{\Phi}' + A_0^{\alpha} \overline{\varphi}^{\alpha} - \mathcal{H} + J \cdot \Phi]\right\} \end{aligned} \quad (48)$$

In the above expression, the integrals over E_k^{α} and Π_H are of Gaussian type and hence are easily calculated, giving

$$\int D(E_k^{\alpha}) e^{-i \int d^4x [\frac{1}{2}(E_k^{\alpha})^2 + E_k^{\alpha} F_{0k}^{\alpha}]} = e^{i \int d^4x \frac{1}{2}(F_{k0}^{\alpha})^2} \quad (49)$$

$$\int D(\Pi_H) e^{-i \int d^4x [\frac{1}{2}\Pi_H^2 - \Pi_H \dot{H}]} = e^{i \int d^4x \frac{1}{2}\dot{H}^2} \quad (50)$$

For the integrals over Π_{ν} and Π_l , the integration variables Π_{ν} and Π_l will be changed to $\overline{\nu}$ and \overline{l} . The Jacobian caused by this change, as a constant, may be put in the constant N . On substituting Eqs.(49) and (50) in Eq.(48), it is easy to see that in the functional integral thus obtained, except for the external source terms, the sum of the other terms in the exponent just give the original Lagrangian shown in Eqs.(1)-(9). Thus, we obtain

$$Z[J] = \frac{1}{N} \int D(\Psi') \det M \delta[\chi] e^{i \int d^4x [\mathcal{L} + J \cdot \Psi']} \quad (51)$$

where $\Psi' = (\overline{l}, l, \overline{\nu}, \nu, A_{\mu}^{\alpha}, H)$. When making use of the familiar expression of the determinant^[4]

$$\begin{aligned}\det M &= \int D(\overline{C}, C) e^{i \int d^4x d^4y \overline{C}^\alpha(x) M^{\alpha\beta}(x, y) C^\beta(y)} \\ &= \int D(\overline{C}, C) e^{i \int d^4x \overline{C}^\alpha \partial^\mu (D_\mu^{\alpha\beta} C^\beta)}\end{aligned}\quad (52)$$

and the Fresnel representation for the delta-functional

$$\delta[\partial^\mu A_\mu] = \lim_{\alpha \rightarrow 0} C[\alpha] \exp\left\{-\frac{i}{2\alpha} \int d^4x (\partial^\mu A_\mu^\alpha)^2\right\} \quad (53)$$

where $C[\alpha]$ is a constant, we arrive at

$$Z[J] = \frac{1}{N} \int D(\Psi) \exp\{i \int d^4x (\mathcal{L}_{eff} + J \cdot \Psi)\} \quad (54)$$

where

$$\mathcal{L}_{eff} = \mathcal{L} - \frac{1}{2\alpha} (\partial^\mu A_\mu^\alpha)^2 + \overline{C}^\alpha \partial^\mu (D_\mu^{\alpha\beta} C^\beta) \quad (55)$$

is the effective Lagrangian and the limit $\alpha \rightarrow 0$ is implied in Eq.(56). With the definition of particle fields listed below^[2,3,8]

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (A_\mu^1 \mp i A_\mu^2) \quad (57)$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_w & -\sin \theta_w \\ \sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} A_\mu^3 \\ B_\mu \end{pmatrix} \quad (57)$$

where θ_w is the Weinberg angle

$$C^\pm = \frac{1}{\sqrt{2}} (C^1 \mp i C^2), \quad \overline{C}^\pm = \frac{1}{\sqrt{2}} (\overline{C}^1 \mp i \overline{C}^2) \quad (58)$$

$$\begin{pmatrix} C_z \\ C_\gamma \end{pmatrix} = \begin{pmatrix} \cos \theta_w & -\sin \theta_w \\ \sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} C^3 \\ C^0 \end{pmatrix} \quad (59)$$

and

$$\begin{pmatrix} \overline{C}_z \\ \overline{C}_\gamma \end{pmatrix} = \begin{pmatrix} \cos \theta_w & -\sin \theta_w \\ \sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} \overline{C}^3 \\ \overline{C}^0 \end{pmatrix} \quad (60)$$

the effective Lagrangian in Eq.(54) can be rewritten in the form

$$\mathcal{L}_{eff} = \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_H + \mathcal{L}_{gf} + \mathcal{L}_{gh} \quad (61)$$

where

$$\begin{aligned}\mathcal{L}_g &= -\frac{1}{2} W_{\mu\nu}^+ W^{-\mu\nu} - \frac{1}{4} [Z^{\mu\nu} Z_{\mu\nu} + A^{\mu\nu} A_{\mu\nu}] + M_w^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_z^2 Z^\mu Z_\mu \\ &\quad + ig[(W_{\mu\nu}^+ W^{-\mu} - W_{\mu\nu}^- W^{+\mu})(\sin \theta_w A^\nu + \cos \theta_w Z^\nu) + W_\mu^+ W_\nu^- (\sin \theta_w A^{\mu\nu} + \cos \theta_w Z^{\mu\nu})] \\ &\quad + g^2 \{W_\mu^+ W_\nu^- (\sin \theta_w A^\mu + \cos \theta_w Z^\mu)(\sin \theta_w A^\nu + \cos \theta_w Z^\nu) - W_\mu^+ W^{-\mu} (\sin \theta_w A_\nu + \cos \theta_w Z_\nu)^2 \\ &\quad + \frac{1}{2} [(W_\mu^+)^2 (W_\nu^-)^2 - (W_\mu^+ W^{-\mu})^2]\} \end{aligned} \quad (62)$$

here

$$\begin{aligned}W_\mu^\pm &= (\partial_\mu W_\mu^\pm - \partial_\nu W_\mu^\pm), \quad Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu, \\ A_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu\end{aligned} \quad (63)$$

and

$$M_w = \frac{1}{2}gv, M_Z = M_w / \cos \theta_w \quad (64)$$

$$\begin{aligned} \mathcal{L}_f = & \bar{\nu} i \gamma^\mu \frac{1}{2} (1 - \gamma_5) \partial_\mu \nu + \bar{l} (i \gamma^\mu \partial_\mu - m_l) l + \frac{g}{\sqrt{2}} (j_\mu^- W^{+\mu} + j_\mu^+ W^{-\mu}) - e j_\mu^{em} A^\mu \\ & + \frac{e}{2 \sin 2\theta_w} j_\mu^0 Z^\mu \end{aligned} \quad (65)$$

in which

$$j_\mu^+ = \bar{l} \gamma_\mu \frac{1}{2} (1 - \gamma_5) \nu = (j_\mu^-)^+ \quad (66)$$

$$j_\mu^{em} = \bar{l} \gamma_\mu l \quad (67)$$

$$j_\mu^0 = \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu - \bar{l} \gamma_\mu (1 - \gamma_5) l + 4 \sin^2 \theta_w j_\mu^{em} \quad (68)$$

and

$$m_l = \frac{1}{\sqrt{2}} f_l v \quad (69)$$

$$\mathcal{L}_H = \frac{1}{2} (\partial^\mu H)^2 - \frac{1}{2} m_H^2 H^2 + \frac{g}{4} (W^{+\mu} W_\mu^- + \frac{1}{2 \cos \theta_w} Z^\mu Z_\mu) (H^2 + 2vH) - \frac{f_l}{\sqrt{2}} \bar{l} l H - \lambda v H^3 - \frac{\lambda}{4} H^4 \quad (70)$$

$$\mathcal{L}_{gf} = -\frac{1}{\alpha} \partial^\mu W_\mu^+ \partial^\nu W_\nu^- - \frac{1}{2\alpha} (\partial^\mu Z_\mu)^2 - \frac{1}{2\alpha} (\partial^\mu A_\mu)^2 \quad (71)$$

and

$$\begin{aligned} \mathcal{L}_{gh} = & \bar{C}^- \square C^+ + \bar{C}^+ \square C^- + \bar{C}_z \square C_z + \bar{C}_\gamma \square C_\gamma \\ & - ig \{ (\partial^\mu \bar{C}^+ C^- - \partial^\mu C^- C^+) (\cos \theta_w Z_\mu + \sin \theta_w A_\mu) \\ & + (\partial^\mu \bar{C}^- W_\mu^+ - \partial^\mu \bar{C}^+ W_\mu^-) (\cos \theta_w C_z + \sin \theta_w C_\gamma) \\ & + (\cos \theta_w \partial^\mu \bar{C}_z + \sin \theta_w \partial^\mu \bar{C}_\gamma) (C^+ W_\mu^- - C^- W_\mu^+) \} \end{aligned} \quad (72)$$

The external source terms in Eq.(56) are defined by

$$\begin{aligned} J \cdot \Psi = & J_\mu^- W^{+\mu} + J_\mu^+ W^{-\mu} + J_\mu^z Z^\mu + J_\mu^\gamma A^\mu + JH + \bar{\xi}_l l \\ & + \bar{l} \xi_l + \bar{\xi}_\nu \nu + \bar{\nu} \xi_\nu + \bar{\eta}^+ C^- + \bar{C}^+ \eta^- + \bar{\eta}^- C^+ + \bar{C}^- \eta^+ + \bar{\eta}_z C_z \\ & + \bar{C}_z \eta_z + \bar{\eta}_\gamma C_\gamma + \bar{C}_\gamma \eta_\gamma \end{aligned} \quad (73)$$

The effective Lagrangian shown above is exactly the same as the one given in the Landau gauge which was obtained in our preceding paper^[1]. In the paper, the quantization of the electroweak theory without involving Goldstone bosons was performed in the Lagrangian path-integral formalism by the Faddeev-Popov's approach and/or the Lagrange multiplier method and the quantum theory was given in the general α -gauge. For the quantization carried out in the Lagrangian path-integral formalism, it is necessary to consider the $SU(2) \times U(1)$ gauge-invariance of the Lagrangian given in the unitary gauge and utilize the $SU(2) \times U(1)$ gauge transformations. Nevertheless, in the quantization performed in the Hamiltonian path-integral formalism, as one has seen in this paper, we do not need to consider any gauge transformation and the gauge-invariance of the Lagrangian used. The same result obtained by the both quantizations indicates that the electroweak theory without the Goldstone bosons surely has the original $SU(2) \times U(1)$ gauge symmetry. This feature of the theory is natural because the theory can be written out from the ordinary R_α -gauge theory^[9,10] by making the Higgs transformation to the original Lagrangian and striking off the Goldstone fields from the gauge-fixing terms and the ghost terms in the effective Lagrangian. This procedure was justified in our preceding paper.

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II. APPENDIX A: EQUATIONS OF MOTION

To help understanding the nature of the constraint conditions in Eqs.(10) and (25), we briefly sketch the derivation of equations of motion for the gauge fields. In order to get first order equations, the Lagrangian in Eq.(2) will be recast in the first order form

$$\mathcal{L}_g = \frac{1}{2}F^{a\mu\nu}F_{\mu\nu}^\alpha - \frac{1}{2}F^{a\mu\nu}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\epsilon^{abc}A_\mu^b A_\nu^c) + \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{2}B^{\mu\nu}(\partial_\mu B_\nu - \partial_\nu B_\mu) \quad (\text{A.1})$$

where the variables $F_{\mu\nu}^\alpha, A_\mu^\alpha, B_{\mu\nu}$ and B_μ are all treated as independent. Then, from the stationary condition of the action $S_\lambda = \int d^4x \mathcal{L}_\lambda$ where \mathcal{L}_λ was defined in Eqs.(1), (A.1), (3) and (4), it is not difficult to derive the following equations

$$\partial^\nu F_{\mu\nu}^a = g\epsilon^{abc}F_{\mu\nu}^b A^{c\nu} + \frac{1}{2}g L \gamma_\mu \tau^a L + \frac{1}{4}g^2 \phi^+ \phi A_\mu^a + \frac{1}{4}gg' \phi^+ \tau^a \phi B_\mu - \partial_\mu \lambda^a \quad (\text{A.2})$$

$$\partial^\nu B_{\mu\nu} = -\frac{1}{2}g'(L \gamma_\mu L + 2\bar{l}_R \gamma_\mu l_R) + \frac{1}{4}g'^2 \phi^+ \phi B_\mu + \frac{1}{4}gg' \phi^+ \tau^a \phi A_\mu^a - \partial_\mu \lambda^0 \quad (\text{A.3})$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\epsilon^{abc}A_\mu^b A_\nu^c \quad (\text{A.4})$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (\text{A.5})$$

and the Lorentz condition written in Eq.(10) as well as the equations for the fermions and the Higgs particle which we do not list here for brevity.

Setting $\mu(\nu) = 0$ and $k = 1, 2, 3$, the above equations will be separately written as follows

$$\partial_0 A_k^a = \partial_k A_0^a - F_{k0}^a + g\epsilon^{abc}A_k^b A_0^c \quad (\text{A.6})$$

$$\partial_0 B_k = \partial_k B_0 - B_{k0} \quad (\text{A.7})$$

$$\partial_0 F_{k0}^a = \partial^l F_{lk}^a + g\epsilon^{abc}(F_{k0}^b A_0^c + F_{kl}^b A^c) + \frac{1}{2}g\bar{L}\gamma_k \tau^a L + \frac{1}{4}g^2 \phi^+ \phi A_k^a + \frac{1}{4}gg' \phi^+ \tau^a \phi B_k - \partial_k \lambda^a \quad (\text{A.8})$$

$$\partial_0 B_{k0} = \partial^l B_{lk} - \frac{1}{2}g'(\bar{L}\gamma_k L + 2\bar{l}_R \gamma_k l_R) + \frac{1}{4}gg' \phi^+ \tau^a \phi A_k^a + \frac{1}{4}g'^2 \phi^+ \phi B_k - \partial_k \lambda^0 \quad (\text{A.9})$$

$$F_{kl}^a = \partial_k A_l^a - \partial_l A_k^a + g\epsilon^{abc}A_k^b A_l^c \quad (\text{A.10})$$

$$B_{kl} = \partial_k B_l - \partial_l B_k \quad (\text{A.11})$$

$$\partial^k F_{k0}^a = g\epsilon^{abc}F_{k0}^b A^{ck} - \frac{1}{2}g\bar{L}\gamma_0 \tau^a L - \frac{1}{4}g^2 \phi^+ \phi A_0^a - \frac{1}{4}gg' \phi^+ \tau^a \phi B_0 + \partial_0 \lambda^a \quad (\text{A.12})$$

$$\partial^k B_{k0} = \frac{1}{2}g'(\bar{L}\gamma_0 L + 2\bar{l}_R \gamma_0 l_R) - \frac{1}{4}gg' \phi^+ \tau^a \phi A_0^a - \frac{1}{4}g'^2 \phi^+ \phi B_0 + \partial_0 \lambda^0 \quad (\text{A.13})$$

As we see, in Eqs.(A.6)-(A.9) there are the time-derivatives of the field variables $A_k^a, B_k, F_{k0}^a = E_k^a$ and $B_{k0} = E_k^0$, whereas in Eqs.(A.10)-(A.13), there are no such derivatives. Therefore, Eqs.(A.6)-(A.9) act as the equations of motion, while, Eqs.(A.10)-(A.13) can only be identified with the constraint equations. With the definitions given in Eqs.(12) and (21-24), we see that Eqs.(A.12) and (A.13) are just combined to give the constraint equation written in Eq.(25).

III. APPENDIX B: THE DELTA-FUNCTIONAL REPRESENTATION OF CONSTRAINT EQUATIONS

In this appendix, we take an example to prove the relation shown in Eq.(38). Suppose we have two equations

$$\varphi_1(x, y) = 0 \quad (\text{B.1})$$

$$\varphi_2(x, y) = 0 \quad (\text{B.2})$$

whose solutions are assumed to be (x_s, y_s) . Let us evaluate the integral

$$I = \int dx dy f(x, y) \delta[\varphi_1(x, y)] \delta[\varphi_2(x, y)] \quad (\text{B.3})$$

where $f(x, y)$ is an arbitrary integrable function. This integral may be easily calculated by making the change of the integration variables

$$\varphi_1(x, y) = u_1, \varphi_2(x, y) = u_2 \quad (\text{B.4})$$

Correspondingly, the integration measure will be changed to

$$dx dy = \det \left(\frac{\partial(\varphi_1, \varphi_2)}{\partial(x, y)} \right)^{-1} du_1 du_2 \quad (\text{B.5})$$

Substituting Eqs.(B.4) and (B.5) in Eq.(B.3), we have

$$\begin{aligned} I &= \int du_1 du_2 \det \left(\frac{\partial(\varphi_1, \varphi_2)}{\partial(x, y)} \right)^{-1} \delta(u_1) \delta(u_2) f(x(u_1, u_2), y(u_1, u_2)) \\ &= \det \left(\frac{\partial(\varphi_1, \varphi_2)}{\partial(x, y)} \right)^{-1} f(x(u_1, u_2), y(u_1, u_2))|_{u_1=u_2=0} \end{aligned} \quad (\text{B.6})$$

Noticing

$$x(u_1, u_2)|_{u_1=u_2=0} = x_s, y(u_1, u_2)|_{u_1=u_2=0} = y_s \quad (\text{B.7})$$

we can write

$$I = \det \left(\frac{\partial(\varphi_1, \varphi_2)}{\partial(x, y)} \right)^{-1} \Big|_{x=x_s, y=y_s} f(x_s, y_s) = \int dx dy f(x, y) \det \left(\frac{\partial(\varphi_1, \varphi_2)}{\partial(x, y)} \right)^{-1} \delta(x - x_s) \delta(y - y_s) \quad (\text{B.8})$$

In comparison of this expression with that denoted in Eq.(B.3), it is clear to see that

$$\delta(x - x_s) \delta(y - y_s) = \det \left(\frac{\partial(\varphi_1, \varphi_2)}{\partial(x, y)} \right) \delta(\varphi_1) \delta(\varphi_2) \quad (\text{B.9})$$

For delta-functionals, certainly, we have the same relation, just as shown in Eq.(38)

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